

Curriculum Vitae

HAROLD WILLIAMS

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Education/Employment

- 2020 – Assistant Professor, University of Southern California
- 2017 – 2020 Postdoctoral Fellow, Center for Quantum Mathematics and Physics, University of California, Davis
- 2014 – 2017 RTG Instructor of Mathematics, University of Texas, Austin
- 2014 Ph.D. University of California, Berkeley, Mathematics (advisor: Nicolai Reshetikhin)
- 2008 B.S. University of Texas, Austin, Mathematics (Summa Cum Laude)
- 2008 B.A. University of Texas, Austin, Music (Summa Cum Laude)

Grants/Scientific Honors

- 2022 – NSF CAREER Grant DMS-2143922 (5 years, \$450,000, “Cluster Algebras in Representation Theory, Geometry, and Physics”)
- 2017 – 2021 NSF Standard Grant DMS-1801969 (3 years, \$107,022, “Cluster Algebras in Representation Theory and Symplectic Geometry”)
- 2015 – 2017 NSF Mathematical Sciences Postdoctoral Research Fellowship DMS-1502845 (sponsoring scientist: Andrew Neitzke)
- 2016 NSF Conference Grant DMS-1601967 (PI, Texas Algebraic Geometry Symposium)
- 2014 Kenneth Ribet and Lisa Goldberg Award in Algebra (UC Berkeley Dissertation Award)

Research Interests

Representation theory, algebraic geometry, symplectic geometry, mathematical physics, combinatorics, cluster algebras, integrable systems.

Papers

Published Articles

1. (with D. Treumann, E. Zaslow) *Kasteleyn operators from mirror symmetry*, Selecta Math. New Ser. (2019) 25: 60.
2. (with V. Shende, D. Treumann, E. Zaslow) *Cluster varieties from Legendrian knots*, Duke Math. J. 168 (2019), no. 15, 2801-2871.
3. (with S. Cautis) *Cluster theory of the coherent Satake category*, J. Amer. Math. Soc. 32 (2019), 709-778.
4. (with D. Rupel, S. Stella) *Affine cluster monomials are generalized minors*, Compositio Math. 155 (2019), 1301-1326.
5. (with D. Rupel, S. Stella) *On generalized minors and quiver representations*. Int. Math. Res. Not. IMRN (2018), doi.org/10.1093/imrn/rny053.
6. (with M. Cheung, M. Gross, G. Muller, G. Musiker, D. Rupel, S. Stella) *The greedy basis equals the theta basis*, J. Combin. Theory Ser. A 145 (2017), 150-171.
7. *Toda systems, cluster characters, and spectral networks*, Commun. Math. Phys. (2016), 1-41.
8. *Cluster characters and the combinatorics of Toda systems*, Theoret. and Math. Phys. 185 (2015), no. 3, 1789-1802.

9. *Q-Systems, factorization dynamics, and the twist automorphism*. Int. Math. Res. Not. IMRN (2015), no. 22, 12042-12069.
10. *Cluster ensembles and Kac-Moody groups*, Adv. Math. 247 (2013), 1-40.
11. *Double Bruhat cells in Kac-Moody groups and integrable systems*, Lett. Math. Phys. 103 (2013), no. 4, 389-419.

Preprints

12. (with S. Cautis) *Canonical bases for Coulomb branches of $4d \mathcal{N} = 2$ gauge theories*, arXiv:2306.03023.
13. (with S. Cautis) *Tamely presented morphisms and coherent pullback*, arXiv:2306.03119.
14. (with S. Cautis) *Ind-geometric stacks*, 2306.03043.
15. (with V. Shende, D. Treumann) *On the combinatorics of exact Lagrangian surfaces*, arXiv:1603.07449.

Articles In Preparation

16. (with T. Gannon) *Differential operators on SL_n/U and quantized Coulomb branches*, expected 2023.
17. (with S. Cautis) *Clifford algebras and linear Koszul duality*, expected 2024.
18. (with C. Kuo) *Bipartite graphs in T^n and toric mirror symmetry*, expected 2024.

Expository Writing

19. *Notes on derived categories as higher categories*, www.hwilliams.net/derivedhigher.pdf.

Conferences Organized

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| 2023 | June | 2023 Los Angeles Workshop on Representations and Geometry, USC |
| 2022 | June | 2022 Los Angeles Workshop on Representations and Geometry, USC |
| 2011 | Oct. | Representation Theory, Geometry, and Combinatorics RTG Conference, UC Berkeley |
| 2016 | April | Texas Algebraic Geometry Symposium, UT Austin |

Invited Conference and Seminar Talks

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| 2023 | Nov | Kansas State University (M-Seminar): Differential operators on the base affine space and quantized Coulomb branches |
| — | Nov | UC Berkeley (Geometry and String Theory Seminar): Differential operators on the base affine space and quantized Coulomb branches |
| — | Oct | UT Austin (Geometry Seminar): Differential operators on the base affine space and quantized Coulomb branches |
| — | Jul | University of Oregon (Workshop on Algebra and Representation Theory on Oregonian Grounds): Categorized Coulomb Branches |
| — | Jun | CIRM, Trento, Italy (“Cluster algebra and Poisson geometry” Conference): Tamely presented algebraic geometry |
| — | Mar | UC Riverside (Lie Theory Seminar): Categorized Coulomb Branches |
| 2022 | Jun | Yale University (“On the Crossroads of Algebra, Geometry and Physics” Conference): Canonical Bases for Coulomb Branches |
| 2021 | Feb | Virtual (UCLA Topology Seminar): Kasteleyn Operators from Mirror Symmetry |
| 2020 | Dec | Virtual (CIRM Conference on Quantum Groups and Cohomology Theories of Quiver and Flag Varieties): Canonical Bases for Coulomb Branches |
| — | Dec | Virtual (Western Hemisphere Colloquium on Geometry and Physics): Canonical Bases for Coulomb Branches |

- Aug Virtual (Dimers in Combinatorics and Cluster Algebras Conference): Kasteleyn Operators from Mirror Symmetry
- 2019 Nov Rutgers University: Geometry and Representation Theory Through the Lens of Physics
- Nov UCSD: Geometry and Representation Theory Through the Lens of Physics
- Nov USC: Geometry and Representation Theory Through the Lens of Physics
- Nov AMS Western Sectional Meeting, UC Riverside (Special Session on Geometric Methods in Representation Theory): Canonical Bases for Coulomb Branches
- Nov AMS Western Sectional Meeting, UC Riverside (Special Session on Canonical Bases, Cluster Structures and Non-commutative Birational Geometry): Kasteleyn Operators from Mirror Symmetry
- Oct Universität Bonn (Representation Theory Seminar): Canonical Bases for Coulomb Branches
- Oct Center for Mathematical Physics, Hamburg (Colloquium): Canonical Bases for Coulomb Branches
- Sept Perimeter Institute for Theoretical Physics (Mathematics Seminar): Canonical Bases for Coulomb Branches
- Sept Giens, France (Conference on Integrability, Combinatorics, and Representations): Kasteleyn Operators from Mirror Symmetry
- Aug Centre de Recherches Mathématiques, Montréal (Quiver Varieties and Representation Theory): Representation Theoretic Interpretations of Acyclic Cluster Algebras
- Jun RIMS, Kyoto (Workshop on Cluster Algebras, Geometry, and Mathematical Physics): Canonical Bases for Coulomb Branches
- Jun International Centre for Mathematical Sciences, Edinburgh (Conference on Geometric Representation Theory, Topological Field Theory, and Low-dimensional Topology): Canonical Bases for Coulomb Branches
- May Centre de Recherches Mathématiques, Montréal (Faces of Integrability): Kasteleyn Operators from Mirror Symmetry
- Apr Kavli IPMU, Tokyo: Canonical Bases for Coulomb Branches
- Feb UT Austin (Texas Algebraic Geometry Symposium): Kasteleyn Operators from Mirror Symmetry
- 2018 Dec University of Edinburgh (Algebra Seminar): Kasteleyn Operators from Mirror Symmetry
- Nov Mathematisches Forschungsinstitut Oberwolfach (Enveloping Algebras and Geometric Representation Theory): The Coherent Satake Category
- Oct Columbia University (Symplectic Geometry, Gauge Theory, and Categorification Seminar): Kasteleyn Operators from Mirror Symmetry
- Oct Northwestern University (Geometry and Physics Seminar): Kasteleyn Operators from Mirror Symmetry
- Jun Notre Dame University (Conference on Geometric Representation Theory and Symplectic Varieties): The Coherent Satake Category
- Jun Université Paris Diderot (3 lecture minicourse): Cluster Theory of the Coherent Satake Category
- Jun CIRM, Luminy (Representation Theory, Mathematical Physics, and Integrable Systems): The Coherent Satake Category
- May Michigan State University (Workshop on Cluster Algebras and Mathematical Physics): The Coherent Satake Category
- Mar CIRM, Luminy (Cluster Algebras: Twenty Years On): The Coherent Satake Category
- Jan Casa Matemática Oaxaca (Workshop on Cluster Varieties and Mathematical Physics): Renormalized R-matrices and Chiral Categories [3 lectures]

- 2017 Nov UNAM, Mexico City (US-Mexico Meeting on Representation Theory, Categorification, and Noncommutative Algebra): Affine Cluster Monomials are Generalized Minors
- Nov University of Toronto (Geometric Representation Theory Seminar): The Coherent Satake Category
- Nov AMS Western Sectional Meeting, UC Riverside (Special Session on Non-commutative Birational Geometry, Cluster Structures, and Canonical Bases): Quiver Representations and Generalized Minors
- Mar Johannes Gutenberg-Universität, Mainz (Spring School on Cluster Algebras in Mathematical Physics): Microlocal Sheaves and Mirror Symmetry for Toric Varieties
- Mar Institut Henri Poincaré (Trimester on Combinatorics and Interactions, Research Seminar): From Dimer Models to Integrable Systems via Mirror Symmetry
- Mar Institut de Mathématiques de Jussieu (Algebra Seminar): Quiver Representations and Generalized Minors
- Jan UC Berkeley (String-Math Seminar): Cluster Theory of the Coherent Satake Category
- Jan University of British Columbia (Colloquium): Canonical Bases and Physical Mathematics
- Jan University of British Columbia (Algebraic Geometry Seminar): Cluster Theory of the Coherent Satake Category
- 2016 Dec Perimeter Institute for Theoretical Physics (Mathematics Seminar): Cluster Theory of the Coherent Satake Category
- Nov University of Virginia (Algebra Seminar): Cluster Theory of the Coherent Satake Category
- Nov UCLA (Colloquium): The Affine Grassmannian as an Object of Symplectic Geometry
- Nov USC (Algebra Seminar): The Affine Grassmannian as an Object of Symplectic Geometry?
- Oct Kavli IPMU, Tokyo: The Coherent Satake Category and Line Operators
- Oct RIMS, Kyoto (Differential Geometry Seminar): Skeletal Surgery and the Combinatorics of Exact Lagrangian Surfaces
- Oct RIMS, Kyoto (Geometric Representation Theory Conference): Cluster Theory of the Coherent Satake Category
- July ETH Zürich (Poisson 2016): Cluster Algebra as the Moduli Theory of A-branes in 4-manifolds
- May Perimeter Institute for Theoretical Physics (Mathematics Seminar): Cluster Theory is the Moduli Theory of A-branes in 4-manifolds
- May Notre Dame London Global Gateway (Quivers and Bipartite Graphs Conference): Clusters are Exact Lagrangian Surfaces
- Mar MSRI (Hot Topics: Cluster Algebras and Wall-Crossing Workshop): Cluster Algebras and Exact Lagrangian Surfaces
- Mar UC Berkeley (Simons Collaboration Conference on Mirror Symmetry and Wall-Crossing): Skeletal Surgery and the Combinatorics of Exact Lagrangian Surfaces
- Jan Simons Center for Geometry and Physics (Geometric Representation Theory Workshop): Microlocal Sheaves and Cluster Algebras
- 2015 Nov UT Austin (Geometry Seminar): Moduli Spaces of Microlocal Sheaves and Cluster Combinatorics
- Nov University of British Columbia (Algebraic Geometry Seminar): Moduli Spaces of Microlocal Sheaves and Cluster Combinatorics
- Oct Northwestern University (Geometry and Physics Seminar): Moduli Spaces of Microlocal Sheaves and Cluster Combinatorics
- Sept Texas A&M University (Texas Algebraic Geometry Symposium Workshop): Moduli Spaces of Microlocal Sheaves and Cluster Combinatorics
- Sept Notre Dame (Felix Klein Seminar): Cluster Combinatorics of Symplectic 4-Manifolds and Legendrian Knots

- July Université de Montréal (CRM Workshop on Positive Grassmannians and Applications): Bipartite Graphs and Microlocal Geometry
- June University of Minnesota (Sage Days 64.5): Cluster Algebras IV
- May Woods Hole Oceanographic Institute (Maurice Auslander International Conference): Cluster Algebras as Legendrian Knot Invariants
- May MIT (Lie Theory Seminar): Cluster Algebras as Microlocal Knot Invariants
- Apr Northeastern University (Geometry and Physics Seminar): Legendrian Knots and Cluster Varieties
- Apr Boston College (Geometry Seminar): Legendrian Knots and Cluster Varieties
- Mar AMS Central Sectional Meeting, Michigan State University (Special Session on Integrable Combinatorics): Legendrian Knots and Cluster Varieties
- Jan UNAM, Mexico City (US-Mexico Meeting on Noncommutative Algebra and Representation Theory): Toda Systems, Cluster Characters, and Spectral Networks
- Jan Joint Mathematics Meetings, San Antonio (Special Session on Cluster Algebras): Toda Systems, Cluster Characters, and Spectral Networks
- 2014 Dec Korea Institute for Advanced Study (Workshop on Strings, Quivers, and Cluster Algebras in Mathematical Physics): Toda Systems, Cluster Characters, and Spectral Networks
- Nov Perimeter Institute for Theoretical Physics (Workshop on Mathematical Physics): Toda Systems, Cluster Characters, and Spectral Networks
- Oct University of California, Berkeley (Geometry and Representation Theory Seminar): Cluster Integrable Systems, Quiver Representations, and Line Operators
- Sept Kavli IPMU, Tokyo: Relativistic Integrable Systems, Quiver Representations, and Line Operators
- Sept Nagoya University (Summer School on Cluster Algebras and Mathematical Physics): Toda Systems, Quiver Representations, and $\mathcal{N} = 2$ Field Theory [3 lectures]
- June Independent University of Moscow (Workshop on Combinatorics of Moduli Spaces, Cluster Algebras, and Topological Recursion) Integrable Systems, Canonical Bases, and $\mathcal{N} = 2$ Gauge Theory
- May Northwestern University (Workshop on Representation Theory, Integrable Systems, and Quantum Fields): Integrable Systems, Canonical Bases, and $\mathcal{N} = 2$ Field Theory [3 lectures]
- May University of Hong Kong: Integrable Systems, Canonical Bases, and $\mathcal{N} = 2$ Field Theory [3 lectures]
- Apr Northwestern University (Geometry and Physics Seminar): Integrable Systems and Canonical Bases
- Feb Stanford University (Workshop on Lie Groups, Lie Algebras, and their Representations): Quivers and Toda Systems
- Jan University of Texas, Austin (Geometry and Strings Seminar): Seiberg-Witten Curves and Double Bruhat Cells
- 2013 Dec Mathematisches Forschungsinstitut Oberwolfach (Workshop on Cluster Algebras and Related Topics): Q -Systems, Double Bruhat Cells, and $\mathcal{N} = 2$ Yang-Mills
- Nov University of Illinois, Urbana-Champaign: Q -Systems, Factorization Dynamics, and Twisting
- Oct University of Bonn: Q -Systems, Factorization Dynamics, and Twisting
- Oct University of Strasbourg: Q -Systems, Factorization Dynamics, and Twisting
- Sept University of California, Berkeley (Representation Theory Seminar): Q -Systems, Factorization Dynamics, and Twisting
- June UNAM, Mexico City (Workshop on Cluster Algebras and BPS Invariants): Seiberg-Witten Curves and Cluster Algebras [3 lectures]
- 2012 Oct Tulane (AMS Southeastern Sectional Meeting): Cluster Ensembles and the Chamber Ansatz

- Sept Aarhus University (Algebra Seminar): Cluster Ensembles and the Chamber Ansatz
- July Maui (Subfactors in Maui Conference): From Planar Algebras to Loop Groups via Triple Crossings
- Apr University of California, Berkeley (Combinatorics Seminar): Loop Groups and Cluster Integrable Systems

Teaching Activities

PhD Students

1. Jishnu Bose (expected 2024)
2. Tina Peng (expected 2025)
3. Tianle Liu (expected 2026)

USC

- 2023 Spring Theory of Numbers (Math 430)
- 2022 Fall Algebraic Topology (Math 540)
- 2022 Spring Vector Analysis and Introduction to Differential Geometry (Math 435)
- 2021 Fall Topics in Topology (Math 641)
- 2021 Spring Vector Analysis and Introduction to Differential Geometry (Math 435)
- 2020 Fall Topics in Algebraic Geometry (Math 614)

UT Austin

- 2015 Spring Differential Calculus (Math 408K)
- 2014 Fall Differential and Integral Calculus II (Math 408D)

UC Berkeley (Graduate Student Instructor)

- 2011 Spring Analytic Geometry and Calculus (Math 16A)
- 2010 Spring Calculus II (Math 1B)
- 2009 Fall Calculus I (Math 1A)
- 2009 Spring Calculus II (Math 1B)
- 2008 Fall Calculus II (Math 1B)

Other Mentorship

- 2014 June Teaching Assistant for AMS Mathematics Research Community on Cluster Algebras, Snowbird, UT

Service Activities

1. Served on qualifying exam committees of Grant Bowling, Sam Armon, Jian Zhou, Haoyang Liu, Wenhan Jiang, and Alex Sahakian.
2. Served on Geometry/Topology Screening Exam committee (3 times).
3. Served on Math Department Graduate Committee since Spring 2022.
4. Served as a co-organizer of the USC Topology Seminar and of the Los Angeles Joint Topology Seminar (Spring 2023).
5. Served on two NSF grant review panels.

Teaching Awards

2011 Spring Outstanding Graduate Student Instructor (UC Berkeley)

Scientific Consulting

2017 – 2020 Research consultant for Helm.ai on machine learning and perception algorithms for autonomous driving.

Journals Refereed

Advances in Mathematics
Algebra and Number Theory
Communications in Mathematical Physics
Compositio Mathematica
Comptes Rendus Mathematique
Duke Mathematical Journal
International Mathematics Research Notices
Inventiones Mathematicae
Journal of the American Mathematical Society
Journal of Integrable Systems
Journal of Physics A
SIGMA